Asymptotic Theory of Particle Trapping in Coherent Nonlinear Alfvén Waves

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A fully nonlinear, time-asymptotic theory of resonant particle trapping in large-amplitude quasi-parallel Alfvén waves is presented. The effect of trapped particles on the nonlinear dynamics of quasi-stationary Alfvénic discontinuities and coherent Alfvén waves is highly non-trivial and forces to a significant departure of the theory from the conventional DNLS and KNLS equation models. The virial theorem is used to determine the time-asymptotic distribution function.

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The magnetic fluctuations frequently observed in Solar Wind and Interstellar Medium plasma have been the subject of protracted and intense observational and theoretical scrutiny. It is likely that these fluctuations are nonlinear Alfvén waves, in which the ponderomotive coupling of Alfvénic magnetic field energy to ion-acoustic quasi-modes has modulated the phase velocity v_A, and so caused steepening and formation of discontinuities [1–3]. Such rotational and directional discontinuities have indeed been observed in the Solar Wind, and are probably quasi-stationary waveform remnants of nonlinearly evolved Alfvén waves [4].

Beginning with the work of Cohen and Kulsrud [5], the theory of quasi-parallel, nonlinear Alfvén waves has received a great deal of attention [6] and has spawned in a variety of modifications of the wave envelope evolution equation, referred to as the Derivative Nonlinear Schrödinger (DNLS) equation. However, almost all attention has been concentrated on developing and extending the fluid theory of such waves, leaving issues of particle kinetics aside. Nevertheless, some attempts to incorporate particle dynamics into the DNLS model have been made, both analytically [7] and (very extensively) via particle- and hybrid-code simulations [8]. Progress in constructing an analytical kinetic-MHD model of nonlinear coherent Alfvén waves occured recently by the self-consistent inclusion of linear Landau damping [2,3] and gyro-kinetic (e.g., ion-cyclotron) effects [3]. However, even in these treatments, wave-particle resonant interaction is treated perturbatively and calculated using the linear particle propagator. This technique fails for a large-amplitude wave propagating in a finite-β plasma (here β is the ratio of kinetic and magnetic pressure) because of non-perturbative effects associated with particle trapping in the field of the wave. In this Letter, we extend the theory of 'kinetic' nonlinear Alfvén waves to the strongly nonlinear regime where trapped particles are important.

In the finite-β, isothermal regimes typical of the Solar Wind (i.e., $c_s \sim v_A,\ T_e \sim T_i$) at 1 AU, resonant interaction of the plasma with ion-acoustic quasi-modes is a critical constituent of the wave dynamics. The very existence of rotational discontinuities is due to the nonlinear coupling of Alfvén waves to (linear) Landau dissipation [1]. Here, linear Landau dissipation refers to damping calculated perturbatively, assuming a Maxwellian particle distribution function (PDF), and thus with a time-independent rate coefficient. This mechanism enters the Alfvén wave dynamics *nonlinearly* (i.e., in proportional to the magnetic energy density of the wave train) because it enters a functional with the parallel ponderomotive force $\propto \partial_z(\tilde{B}_\perp^2/8\pi)$. The 'kinetic' wave equation, called the Kinetic Nonlinear Schrödinger (KNLS) equation, is [2,3]:

$$\frac{\partial \mathrm{b}}{\partial \tau} + v_A \frac{\partial}{\partial z}\left(m_1 \mathrm{b}|\mathrm{b}|^2 + m_2 \mathrm{b}\, \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\mathcal{P}}{(z'-z)} |\mathrm{b}(z')|^2 \mathrm{d}z' \right) + i\frac{v_A^2}{2\Omega_c}\frac{\partial^2 \mathrm{b}}{\partial z^2} = 0, \tag{1}$$

where $\mathrm{b} = (\tilde{B}_x + i\tilde{B}_y)/B_0$ is the normalized complex wave amplitude, Ω_c is the ion-cyclotron frequency, the coefficients m_1 and m_2 are functions of β and T_e/T_i only (see [3]), and $\mathcal{P}$ means the principal value integration.

Obviously, particles which are near resonance with the wave ($v \simeq v_A$) will be trapped by the ponderomotive potential (or equivalently, by the electrostatic fields of driven ion-acoustic perturbations). Particle bounce motion significantly modifies the PDF near resonance, since trapped particle phase mixing results in flattening of the PDF (for resonant velocities) and formation of a *plateau*. Thus, the *linear* calculation of the Landau dissipation, while correct for times short compared to the typical bounce (trapping) time, $\tau \ll \tau_{tr}$, fails for quasi-stationary waveforms for times $\tau \gtrsim \tau_{NL} \gg \tau_{tr}$ (τ_{NL} is the typical nonlinear wave profile evolution time). Hence, Landau dissipation should be calculated *non-perturbatively* to determine the resonant particle response to the nonlinear wave.

Of course, the nonlinear Landau damping problem is, in general, not *analytically* tractable, as it requires explicit expressions for *all* particle trajectories as a function of initial position and time. Such trajectories cannot be

explicitly calculated for a potential of *arbitrary shape*. Usually, a full particle simulation is required to obtain this information. In some cases, an approximate analytic expression for the wave profile shape is known and may be assumed to persist, while the wave amplitude varies. Calculations defined in this way has been implemented for the special cases of sinusoidal [9] and solitonic [10] wave modulations. Other approaches either seek the asymptotic ($\tau \to \infty$) PDF for a given (undamped) waveform [11], or exploit the universality of the process of de-trapping of resonant particles from a wave potential of decreasing amplitude [12,13]. These approaches, however, do not appear to be useful for the problem considered here.

The goal of this work is to investigate how trapped particles modify nonlinear wave evolution, assuming no restrictions on the shape of the wave-packet modulation. Thus, the motion of particles is treated self-consistently. We show that, in the two important limits of short-time ($\tau \ll \tau_{tr}$) and long-time ($\tau \gg \tau_{tr}$) evolution, the problem admits *analytic* solutions. In the limit $\tau \ll \tau_{tr}$, we recover conventional linear Landau damping. This supports the validity of the KNLS theory as a means for studying the *emergence* of Alfvénic discontinuities. In the opposite limit $\tau \gg \tau_{tr}$, the *virial theorem* is used for determination of the time-asymptotic trapped particle response. Although the damping rate vanishes due to phase mixing, the effects of trapped particles are highly non-trivial, leading to a significant departure of the theory from the familiar form of the DNLS and KNLS models. First, the power of the KNLS nonlinearity associated with resonant particles increases to *fourth* order when trapped particles are accounted for. Second, the effective coupling now is proportional to the *curvature* of the PDF at resonant velocity, $f_0''(v_A)$, and not its slope, $f_0'(v_A)$, as in linear theory. Third, the *phase density* of trapped particles is controlled by the plasma β. Finally, we combine these to obtain the wave evolution equation which governs the *long-time* dynamics of quasi-stationary Alfvénic discontinuities. The equation is the principal result of this Letter.

We should state here that particle trapping may be absent in higher than one dimension. Indeed, for $k_\perp \rho_i \gg 1$ ($k_\perp$ is the perpendicular component of the wave vector and ρ_i is the ion Larmor radius), then the longitudinal Čerenkov resonance $\omega = k_\parallel v_\parallel$ is satisfied for all particles having $v = v_A / \cos\Theta$, but with $v_\perp$ arbitrarily large. Thus, all particles with velocities $v \gtrsim v_A$ interact with a wave and a plateau cannot form, while a non-thermal tail of energetic particles may result instead. However, if the magnitude of the ambient magnetic field is strong enough so that $k_\perp \rho_i \ll 1$, quasi-one-dimensionality is recovered. This last situation is, in fact, typical for waves propagating in the Solar Wind.

For reasons of notational economy, let's introduce the trapping potential $U(z) \equiv \tilde{B}_\perp^2 / 8\pi n_0$, where n_0 is the unperturbed plasma particle density. Then, the characteristic bounce frequency [9] in our case is $\tau_{tr}^{-1} \simeq k\sqrt{U/m_i}$ (m_i is the ion mass). The characteristic nonlinear frequency at which the wave profile changes appreciably is readily estimated from Eq. (1) to be $\tau_{NL}^{-1} \simeq m_1 k v_A (\tilde{B}_\perp^2 / B_0^2)$. From comparison of these two timescales, we conclude that the wave potential, as seen by a trapped particle, is *steady-state* (i.e., roughly constant on the particle bounce time) when $\tau_{NL} \gg \tau_{tr}$, so that

$$\tilde{B}_\perp / B_0 \lesssim m_1^{-1} \sim 1. \tag{2}$$

That is, particle phase mixing is very *efficient* for weakly nonlinear waves. Note, this condition (2) is consistent with the derivation of the KNLS, for which $\tilde{B}_\perp / B_0 \ll 1$ (weak nonlinearity) is assumed. Let's now rewrite Eq. (1) in a generic form:

$$\frac{\partial \mathrm{b}}{\partial \tau} + v_A \frac{\partial}{\partial z}\left(m_1 \mathrm{b}\, \delta n_{NR} + m_2 \mathrm{b}\, \delta n_R \right) + i \frac{v_A^2}{2\Omega_c} \frac{\partial^2 \mathrm{b}}{\partial z^2} = 0. \tag{3}$$

Here δn_{NR} is the density perturbation due to the *non-resonant* (bulk) response of the PDF. It is roughly proportional to $|\mathrm{b}|^2$. δn_R is the *resonant* particle contribution. It is responsible for *strongly nonlinear* feedback via the distortion of the PDF by a wave. It was also responsible for linear damping in the KNLS equation. It is interesting that the very possibility to write the generalized KNLS equation in the form (3) relies on the intrinsic *time reversibility* of the Vlasov equation, linear or nonlinear. Indeed, one can formally write the resonant particle response as

$$\delta n_R \propto \chi_\parallel \hat{\mathcal{K}} \left[U(z) \right], \tag{4}$$

where $\hat{\mathcal{K}}$ is some normalized kinetic operator acting on a wave field. Time reversibility implies $\hat{\mathcal{K}}\hat{\mathcal{K}} = -1$, see [3]. This fact has been crucial for the derivation of KNLS. The constant $\chi_\parallel$ plays a role of effective dissipation coefficient (thermal conductivity) in the linear Landau damping theory.

The resonant particle response is calculated using **Liouville's theorem**, which states that *the local PDF is constant along particle trajectories*:

$$f(v, z, t) = f_0\left(v_\pm \left(E, z_0^\pm \right) \right), \tag{5}$$

where $z_0^\pm = z_0^\pm \left(z, t, E; U(z) \right)$ is the *initial* coordinate of a particle of total energy E which at time t is at the point z and has a velocity $v_\pm(E, z)$. Thus, $z_0^\pm$ is a solution of $t = (\pm) \int_{z_0^\pm}^{z} \left[\frac{2}{m_i} \left(E - U(z) \right) \right]^{-1/2} \mathrm{d}z$. By definition $\delta n_R = \int_{\Delta v_{res}} \mathrm{d}v \left(f - f_0^{(t=0)} \right)$:

$$\delta n_R = \frac{1}{\sqrt{2 m_i}} \sum_{(\pm)} \int_{\Delta E_{res}} \frac{f_0\left(v_\pm \left(E, z_0^\pm \right) \right) - f_0\left(v_\pm (E, z) \right)}{\sqrt{E - U(z)}}\, \mathrm{d}E\ . \tag{6}$$

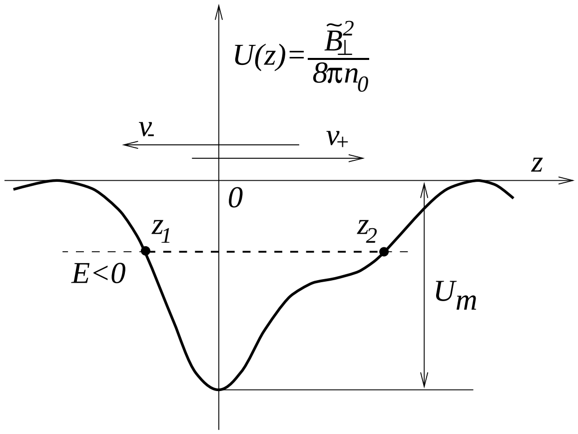

FIG. 1. Trapping potential.

Here the sum is over particles moving to the right (+) and to the left (-), as in Fig. 1. The integration is over the resonant (negative) energies of trapped particles, $U_m \leq E \leq 0$ with U_m being the amplitude of the potential.

Let's first consider the short-time limit, $\tau \to 0$. Then the following approximations are valid. First (i), the effective resonance width [9] is $\Delta E_{res} = (\Delta v_{res})^2 \sim 1/(k\tau)^2 \to \infty$, as $\tau \to 0$. Second (ii), the particle velocity change is negligible $|v(z,E) - v(z_0,E)| \ll \sqrt{U(z)}$, so that (iii), the particle position is roughly proportional to time $z_0^\pm \simeq z \pm \tau\sqrt{2E/m_i}$. Finally (iv), the PDF response can be linearized (in the wave frame moving with v_A) as $f_0(v) \simeq f_0(v_{res}) + v\, f_0'(v_{res})$. Then Eq. (6) may be estimated as

$$\delta n_R\Big|_{t\to 0} \simeq \frac{f_0'(v_A)}{2m_i}\sum_{(\pm)} \pm\int_{U_m}^{\pm\infty}\frac{\mathrm{d}E}{E}\, U\Big(z \pm \tau\sqrt{2E/m_i}\Big)$$
$$\simeq \frac{\pi f_0'(v_A)}{m_i}\left\{\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathcal{P}}{(z'-z)}U(z')\mathrm{d}z'\right\}. \qquad (7)$$

Here, we first used (i) to extend the integration over ΔE_{res} to $\pm\infty$, and then (iii) and (i) to expand the denominator in (6) for $E \gg U$. Finally, we took the $\tau \to 0$ limit. Compared to Eq. (4), the particle operator $\hat{\mathcal{K}}$ is replaced by the Hilbert operator, $\hat{\mathcal{H}}$, given by the expression in curly brackets. It is nonlocal and satisfies the time-reversibility condition $\hat{\mathcal{H}}\hat{\mathcal{H}} = -1$. The effective dissipation coefficient is simply $\chi_\parallel = \pi f_0'(v_A)/m_i$. Thus the KNLS equation (1) is recovered [3].

To treat the $\tau \to \infty$ limit, we recall that for the times $\tau \gtrsim \tau_{NL}$ steady-state waveforms (discontinuities) have formed. Thus, particles are trapped in these adiabatically changing potentials. Hence, we may employ the **virial theorem**, which states that *for any finite motion* in a potential $\tilde{U}(z) = U(z) - U_m$ [i.e., $\tilde{U}(z) \geq 0$] *the (period) averaged* *kinetic* *and* *potential* *energies are related by*

$$2\langle K(z)\rangle = n\langle\tilde{U}(z)\rangle. \qquad (8)$$

Here $\tilde{U}(z)$ is a *homogeneous* function of its argument of order n, i.e., $\tilde{U}(az) = a^n\tilde{U}(z)$. The resonance width is easily estimated to be $\Delta v_{tr} \simeq \sqrt{2|U_m|/m_i}$ with $|U_m| \sim \tilde{B}_\perp^2/8\pi n_0$. Thus, for weak nonlinearity, the resonance is *narrow*:

$$\frac{\Delta v_{tr}}{v_A} \sim \frac{\tilde{B}_\perp}{B_0} \ll 1. \qquad (9)$$

Hence, an expansion of the PDF is valid, so that (in the wave frame): $f_0(v_\pm) \simeq f_0(v_A) \pm v_\pm f_0'(v_A) + \left(v_\pm^2/2\right) f_0''(v_A)$. With this in hand and using Eq. (8) and $\langle U\rangle + \langle K\rangle = E$, we calculate the resonant particle contribution, Eq. (6):

$$\langle\delta n_R\rangle\Big|_{\tau\to\infty} \simeq f_0''(v_A)\sqrt{\frac{2}{m_i^3}}\sqrt{|U(z)|}$$
$$\times\left[\frac{n}{n+2}\left(|U_m| - |U(z)|\right) - \frac{2}{3(n+2)}|U(z)|\right]. \qquad (10)$$

Note that the term $\propto f_0'(v_A)\left[v(z_0^+) - v(z_0^-)\right]$ vanishes identically because $\langle U(z_0^+)\rangle = \langle U(z_0^-)\rangle$. Thus damping is absent. Since $\langle\mathcal{K}\rangle\langle\mathcal{K}\rangle \neq \mathcal{K}\mathcal{K} = -1$, we can, however, only *estimate* [from Eq. (4)] the coupling constant to be $\chi_\parallel \sim f_0''(v_A)\sqrt{2/m_i^3}$. The index n is formally not defined for an arbitrary potential. One may, however, *estimate* it comparing the calculated bounce period in the homogeneous potential and "actual" one determined numerically for a known U, i.e.,

$$T_{hom}(E) = |E|^{\frac{1}{n}-\frac{1}{2}}, \qquad (11a)$$
$$T_{act}(E) = \sqrt{\frac{m_i}{2}}\int_{z_1}^{z_2}\frac{\mathrm{d}z'}{\sqrt{E - U(z')}}. \qquad (11b)$$

It is interesting that the limit $n \to \infty$ encompasses two frequently encountered shapes of a wave packet, namely the *solitonic* and *rectangular* (i.e., deep narrow well) forms. In fact, for these cases as well as for any rather *anharmonic* potentials ($n \gg 2$) the resonant particle response (10) is independent of n and takes on a very simple form:

$$\langle\delta n_R\rangle\Big|_{\substack{\tau\to\infty\\ n\to\infty}} \simeq f_0''(v_A)\sqrt{\frac{2}{m_i^3}}\sqrt{|U(z)|}\left(|U_m| - |U(z)|\right). \qquad (12)$$

Thus, in the long-time limit, $\tau \gg \tau_{tr}$, the damping rate vanishes due to phase mixing. Nevertheless, the resonant particles still contribute the wave dynamics, in that

$$\langle\delta n_R\rangle \sim f_0''(v_A)|\mathrm{b}|^3, \qquad (13)$$

thus determining a new nonlinear wave equation.

To estimate the number of trapped particles, we use a BGK-type (Bernstein-Green-Kruskal) approach [11]. This allows us to find the PDF such that the wave of a

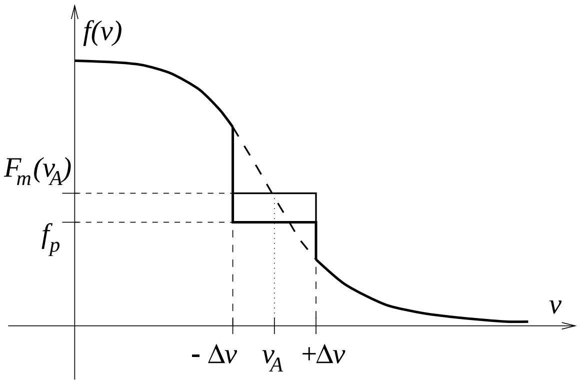

FIG. 2. Asymptotic, $\tau \to \infty$, particle distribution function.

given profile is not dissipated by the Landau mechanism. In our problem there is only one resonance at $v \simeq v_A$, since all modes are coherent. At large times, particle bounces result in flattening of the PDF at resonance so that $f_0'(v_A) \to 0$. The height of this plateau (i.e., the phase density of trapped particles) depends on the wave evolution at earlier times. We take the "unperturbed" (trial) PDF as superimposed plateau and Maxwellian, as in Fig. 2:

$$f_0(v) = F_m(v) + [f_p - F_m(v)]\ \Theta_{v_A}(\Delta v)\,, \tag{14}$$

where $F_m(v)$ is Maxwellian, the Θ-function is defined as

$$\Theta_{v_A}(\Delta v) = \begin{cases} 1, & \text{if } (v_A - \Delta v) \le v \le (v_A + \Delta v); \\ 0, & \text{otherwize;} \end{cases}$$

and f_p is the constant to be determined. The coefficient f_p has a simple meaning of the phase density of trapped particles after the plateau has been formed. Thus, the state with $f_p > F_m(v_A)$ corresponds to a *clump* on the PDF and that with $f_p < F_m(v_A)$ corresponds to a *hole*. The kinetic equation for a perturbation of the PDF is:

$$\left(-i\omega_k + \gamma_k + ik_\parallel v_\parallel\right)\tilde{f}_{\omega,k} = ik_\parallel U_{k_\parallel}\frac{\partial f_0(v)}{\partial v_\parallel}\ . \tag{15}$$

By definition, $\delta n_{\omega,k} = \int f_{\omega,k}\mathrm{d}v$. Then, for $\gamma_k \ll \omega_k = k_\parallel v_A$ and $\Delta v/v_A \ll 1$, we obtain:

$$\delta n_R = \sum_k e^{ik_\parallel z}U_{k_\parallel}\ 2k_\parallel^2\Delta v \times \frac{-\left(i\gamma_k/k_\parallel\right)F_m'(v_A) + [F_m(v_A) - f_p]}{\gamma_k^2 + k_\parallel^2\Delta v^2}\ . \tag{16}$$

Looking for the stationary solution, $\gamma_k = 0$, of the general KNLS equation (3) and neglecting dispersion, we have $\partial_z[\mathrm{b}\delta n_{NR} + \mathrm{b}\delta n_R] = 0$. Consequently,

$$m_1\mathrm{b}|\mathrm{b}|^2 + m_2\mathrm{b}\sum_k e^{ik_\parallel z}|\mathrm{b}|^2_{k_\parallel}\frac{F_m(v_A) - f_p}{\Delta v/2} = 0\ .$$

We thus obtain the trapped particle phase density:

$$f_p = F_m(v_A) + \frac{m_1}{m_2}\frac{\Delta v}{2v_A^2}\ , \tag{17}$$

with $\Delta v \equiv \Delta v_{tr} \simeq v_A(\tilde{B}_\perp/B_0)$. $F_m(v_A)$ is the particle phase density in the absence of trapping. Recalling that m_1 and m_2 are functions of β and $\chi_\parallel \propto f_0'(v_A)$ [3], we conclude that there must be an *under-population* of trapped particles $[f_0 < F_m(v_A)]$ in a low-β plasma ($\beta \lesssim 1$) and an *over-population* $[f_0 > F_m(v_A)]$ in a high-β plasma ($\beta \gtrsim 1$).

Finally, consider the there is weak wave damping not associated with Čerenkov resonance (e.g., as in ion-cyclotron or collisional damping). Then the wave amplitude will slowly decrease, keeping resonant particles trapped. The following adiabatic invariant is thus conserved:

$$J = \oint p_\parallel \mathrm{d}z \simeq const, \tag{18}$$

i.e., $\langle|v_\parallel|\rangle(z_2 - z_1) \simeq const$. From Eq. (1), one can estimate $\Delta z \sim (\Omega_c/v_A)(\tilde{B}_\perp/B_0)^{-2}$. Hence, $\Delta v_\parallel \sim (\tilde{B}_\perp/B_0)^2$. The resonance width is, however, $\Delta v_{tr} \sim (\tilde{B}_\perp/B_0)$. Thus,

$$\frac{\Delta v_\parallel}{\Delta v_{tr}} \sim \frac{\tilde{B}_\perp}{B_0}, \tag{19}$$

that is, the trapped particles will *condense* near the bottom of the potential well, as the wave amplitude decreases. This results in a decrease in the effective index n, which approaches the asymptotic limit $n \to 2$. The BGK analysis given above is, however, then no longer applicable. It should be emphasized that trapped particles condense in the bottom of the potential, rather than de-trap from it, as naively suggested in Ref. [12]. Thus, no asymptotic, power-law damping exists in this case. Obviously, our considerations above are rather generic and valid for a wide class of nonlinear wave systems with quadratic nonlinearity and higher, and thus call the validity of the results of Ref. [12] into general question.

To conclude, we have shown that the effects of the nonlinear PDF modification by a high-amplitude Alfvén wave significantly modify the dynamics of such a wave. Even when phase mixing is efficient enough to quench linear Landau dissipation, trapped particles produce finite a response which modifies the wave nonlinearity. The equation which explicitly describes the evolution of *quasi-stationary* Alfvénic discontinuities and asymptotic ($\tau \to \infty$) dynamics of nonlinear Alfvén waves, Eqs. (3, 10), has been obtained. this result constitutes the extension of the well established DNLS-KNLS theory of quasi-parallel nonlinear Alfvén waves to the strongly nonlinear regime of particle trapping. The phase density of trapped particles has been shown to be controlled by the value of plasma β, as well as wave amplitude.

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[13] We want to comment here that the theory of Ref. [12] is, probably, not as general as it is claimed, and the problem considered in this Letter is, in fact, one of the counter-examples; see end of this Letter.